

MATH 141: Quiz 7

Name: Key

Directions:

- * Show your thought process (commonly said as "show your work") when solving each problem for full credit. **Remember to fully simplify.**
- * If you do not know how to solve a problem, try your best and/or explain in English what you would do.
- * Good luck!

1. Given

$$f''(x) = 20x^3 + 12x^2 + 4, \quad f(0) = 8, \quad f(1) = 5$$

Find the function $f(x)$.

Use general antiderivatives.

$$\begin{aligned} f'(x) &= 20 \cdot \frac{x^4}{4} + 12 \cdot \frac{x^3}{3} + 4x + C \\ &= 5x^4 + 4x^3 + 4x + C \end{aligned}$$

$$\begin{aligned} f(x) &= 5 \frac{x^5}{5} + 4 \frac{x^4}{4} + 4 \frac{x^2}{2} + Cx + D \\ &= x^5 + x^4 + 2x^2 + Cx + D \end{aligned}$$

Use initial conditions to find C and D

$$8 = f(0) = 0^5 + 0^4 + 2 \cdot 0^2 + C \cdot 0 + D$$

So $D = 8$. Substitute in.

$$5 = f(1) = 1^5 + 1^4 + 2 \cdot 1^2 + C \cdot 1 + 8$$

$$5 = 12 + C \rightarrow C = -7$$

$$f(x) = x^5 + x^4 + 2x^2 - 7x + 8$$

2. Integrate the following

$$(a) \int_{-2}^{21} 3 dx = 3x \Big|_{-2}^{21} = 3 \cdot 21 - 3 \cdot (-2) = \boxed{69}$$

$$(b) \int_1^3 (x+1) dx = \frac{x^2}{2} + x \Big|_1^3 = \frac{3^2}{2} + 3 - \left(\frac{1^2}{2} + 1 \right) \\ = \frac{9}{2} + 3 - \frac{3}{2} \\ = \boxed{6}$$

$$(c) \int_{-2}^2 (4x^3 - x^2 + 1) dx = 4 \frac{x^4}{4} - \frac{x^3}{3} + x \Big|_{-2}^2 \\ = x^4 - \frac{x^3}{3} + x \Big|_{-2}^2 \\ = 2^4 - \frac{2^3}{3} + 2 - \left((-2)^4 - \frac{(-2)^3}{3} - 2 \right) \\ = 18 - \frac{8}{3} - \left(14 + \frac{8}{3} \right) \\ = 4 - \frac{16}{3} \\ = \boxed{-\frac{4}{3}}$$